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ESTI CONTROL NR. **AL 45139**CY NR. 1 OF 1 CYS**Group Report****1965-7****Smoothing and Processing  
of Simulated  
AMRAD Trajectory Data****A. Bertolini  
S. F. Catalano****21 January 1965**Prepared for the Advanced Research Projects Agency  
under Electronic Systems Division Contract AF 19(628)-500 by**Lincoln Laboratory**

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Lexington, Massachusetts



ADD 612151

The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology. This research is a part of Project DEFENDER, which is sponsored by the U.S. Advanced Research Projects Agency of the Department of Defense; it is supported by ARPA under Air Force Contract AF 19(628)-500 (ARPA Order 498).

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
LINCOLN LABORATORY

SMOOTHING AND PROCESSING  
OF SIMULATED AMRAD TRAJECTORY DATA

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*Group 45*

GROUP REPORT 1965-7

21 JANUARY 1965



## ABSTRACT

In this report, an attempt is made to determine suitable smoothing intervals to be used in the smoothing of radar trajectory data. This is done by using different fitting intervals in the smoothing of simulated noisy trajectory data of different signal-to-noise ratios. The errors, or differences between the true noiseless data and the smoothed noisy data, were observed and plotted. Minimum errors were obtained when the range smoothing interval was approximately 1 to 2 seconds and the elevation and azimuth smoothing intervals 10 seconds. The plots suggest that elevation and azimuth errors can be further reduced by smoothing over intervals longer than 10 seconds but this would not be practical in many actual missile shots.

Accepted for the Air Force  
Stanley J. Wisniewski  
Lt Colonel, USAF  
Chief, Lincoln Laboratory Office



## INTRODUCTION

An attempt has been made to ascertain appropriate fitting intervals for the smoothing of AMRAD-type trajectory data. It is felt that the results obtained will be useful in processing data from ATHENA shots.

## DESCRIPTION

A noiseless trajectory was generated (20 points/sec) to simulate a typical AMRAD shot. Different noise levels\* were added to this basic trajectory so as to simulate the following signal-to-noise ratios: 0 db, 5 db, 10 db,  $\infty$  db (noiseless). The resultant noisy trajectory data was then smoothed, separately in range, elevation, and azimuth, by standard quadratic least squares procedures of the "sliding arc" type and the following quantities were obtained: range, range rate, range acceleration (rate of range rate), elevation, elevation rate, azimuth, azimuth rate, and drag velocity. The corresponding noiseless trajectory values were subtracted from these and the differences, or errors, were plotted versus time. The above procedure was repeated using different smoothing intervals and the stated signal-to-noise ratios. Also, the rms errors (i.e., the standard deviation of the differences) were computed, tabulated, and, for the 5 db case, plotted versus smoothing interval. Other parameters such as drag acceleration, weight-to-drag ratio, height, path angle, etc., were computed and plotted versus time for selected cases.

## RESULTS

In Figure 1, the two plots on the left describe the geometry of the

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\*See the Appendix for a description of the noise generation.

simulated trajectory giving latitude versus longitude and height versus surface range with AMRAD located at the origin. No tangent-plane approximations are involved in the height versus surface range plot. That is, height and surface range are measured above and along the surface of a spherical earth. The two figures on the right show range, elevation, and azimuth for the noiseless and  $S/N = 5$  db cases.

Figures 2 through 8 show the various errors (i.e., computed value minus true value) in range, range rate, range acceleration, elevation, elevation rate, azimuth, and azimuth rate for the cases tested. Figure 9 gives the drag velocity error versus time for the cases tested. The "drag velocity"  $V_D$  is the speed of the missile relative to the radar, or equivalently, it is the airspeed, assuming a rigid rotating atmosphere with no local winds.

Plots of the standard deviation (or rms values) of the above errors versus smoothing interval for the case of 5 db signal-to-noise ratio are given in Figures 10 and 11. Tables I through III list the rms errors for all cases tested.

Figure 12 shows weight-to-drag ratio, drag acceleration, drag velocity, height, path angle, and viewing (or aspect) angle versus time for the 5 db signal-to-noise ratio case and different smoothing. The path angle GAM is the angle between the velocity vector and the local horizontal. The viewing (or aspect) angle PSI is the angle between velocity vector and the line of sight. TFTR, TFTE, and TFITA are the smoothing intervals for range, elevation and azimuth respectively. The expressions used for



the computation of all parameters in Fig. 12 are as reported in 47G-6, "Determination of Weight-to-Drag Ratio from Radar Measurements," 8 March 1963, S. F. Catalano, H. Schneider. Equation (33) of the above report was used for computations of weight-to-drag ratio. Note that weight-to-drag ratio is best estimated in regions of high acceleration.

### CONCLUSIONS

Referring to Tables I through III and considering the noiseless case, smallest rms errors were obtained when the shortest smoothing intervals were used. This is as expected. With no noise, smoothing is unnecessary as it only corrupts the signal. As noise is added, the smoothing should suppress the noise more than the signal, thereby improving the signal-to-noise ratio. (This will be true if the noise fluctuates much more rapidly than the signal varies.) With excessively long smoothing intervals, the signal may also be seriously distorted, thus losing any advantage gained by noise suppression.

For a signal-to-noise ratio of 5 db, minimum rms errors were obtained when the range smoothing interval was approximately 1 to 2 seconds, and the elevation and azimuth smoothing intervals 10 seconds. Figure 11 suggests that elevation and azimuth errors can be further reduced by smoothing over intervals longer than 10 seconds, but this would not be practical in many actual missile shots. From Figure 10 it is seen that the minimum errors for range, range rate, and range acceleration do not occur at the same range-smoothing interval. The minima, however, are reasonably close together and quite broad, so that very good results should be obtained using a range smoothing interval from 1 to 2 seconds.

A range smoothing interval within these limits should be quite suitable with other signal-to-noise ratios as may be seen from Table I.

Drag velocity and weight-to-drag ratio are used as initial conditions in a trajectory prediction program. Figures 9 and 12 should be useful in assuring sufficient accuracy of these quantities.

The results in this report should be considered as a useful guide rather than absolute rules.

## APPENDIX

### Discussion of Simulated Trajectory Data Generation

The initial conditions at the start of the generated trajectory were as follows:

Time	400.000 seconds
Range	499.791 kft
Range Rate	-20.082 kft/sec
Elevation	31.464 degrees
Elevation Rate	.016 degrees/second
Azimuth	-25.00 degrees
Azimuth Rate	0.0 degrees/second
Weight-to-Drag Ratio	110 lbs/ft <sup>2</sup>

From the above conditions, the starting altitude was 265.160 kft, and the drag velocity 20.082 kft/sec. A trajectory generation program, using a spherical rotating earth with gravity and a realistic atmosphere, calculated range, azimuth, and elevation for the next 999 points spaced .05 seconds apart. At the last calculated data point, the range and altitude were 87.359 kft and 33.113 kft respectively. Gaussian noise, consistent with the desired signal-to-noise ratio was then added to the trajectory data and the resulting noisy data were written on magnetic tape.

The standard deviation for the Gaussian noise added to the signal was computed as described below. (These formulas are based on the assumption of a large signal-to-noise ratio. While this was not always the case here, it was felt that useful results would still be obtained).

For range data\*

$$\sigma_R = \left(\frac{C}{2}\right) \delta T_R = \left(\frac{C}{2}\right) \frac{t_r}{2(S/N)}$$

where  $\sigma_R$  = rms range error

$C$  = velocity of light =  $9.83514 \times 10^8$  ft/sec

$\delta T$  = rms error in estimating leading edge of pulse

$t_r$  = rise time of pulse

$S/N$  = signal-to-noise (power) ratio

The pulse rise time was assumed to be 35 nsec corresponding to a signal bandwidth of approximately 20 Mcps. For elevation and azimuth data\*\*

$$\sigma_\theta = \frac{0.628 \theta_B}{2(S/N)}$$

where  $\sigma_\theta$  = rms elevation (or azimuth) error

$\theta_B$  = antenna beamwidth in elevation (or azimuth)

$S/N$  = signal-to-noise (power) ratio

The units of  $\sigma_\theta$  will be identical to those of  $\theta_B$ . For the simulations  $\theta_B$  was taken as  $1^\circ$  in both azimuth and elevation.

Since the noise bandwidth of the AMRAD antenna servo system is approximately 1 cps, the noise samples to be added to azimuth (or elevation) data would not be independent from point to point with a rep rate of 20 points/sec. The following formula was used to generate the azimuth (and elevation) noise.

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\*Skolnik, M. I., "Introduction to Radar Systems," McGraw-Hill Book Company, 1962, pp. 462-464.

\*\*Skolnik, op. cit., pp. 476-477. 6

$$n_k = \sum_{j=k}^{j=k+N-1} m_j$$

$n_k$  = Gaussian noise to be added to  $k^{\text{th}}$  azimuth (or elevation) sample

$m_j = j^{\text{th}}$  independent Gaussian noise sample of standard deviation  $\sigma_\theta$

$$N = \left( \frac{\text{Repetition Rate}}{\text{Noise Bandwidth}} \right)^{1/2}$$

This assured that the noise would be correlated over a 1 second interval.

The calculated and observed rms noise values are given below:

S/N	$\sigma_R$ (feet)		$\sigma_{EL}$ (degrees)		$\sigma_{AZ}$ (degrees)	
	Calculated	Observed	Calculated	Observed	Calculated	Observed
10 db	3.8	3.9	.14	.13	.14	.12
5 db	6.8	6.9	.25	.22	.25	.23
0 db	12.0	11.8	.44	.48	.44	.48

Signal-to-Noise Ratio	Range Smoothing Interval (seconds)	RMS Range Error (feet)	RMS Range-Rate Error (ft/sec)	RMS Range Acceleration Error (ft/sec <sup>2</sup> )
Noiseless	.5	2.0	1.1	3.6
	2.0	1.9	14.4	5.2
	4.0	3.7	55.3	15.5
10 db	2.0	2.1	14.5	6.8
5 db	.5	4.2	13.8	194.6
	1.0	3.4	6.4	39.4
	1.5	3.1	8.8	14.1
	2.0	3.0	14.6	8.6
	4.0	4.3	55.4	15.6
0 db	2.0	3.3	15.9	13.4

TABLE I

RMS Errors of Range, Range Rate, and Range Acceleration for Different Range Smoothing Intervals and Signal-to-Noise Ratios

Signal-to-Noise Ratio	Elevation Smoothing Interval (seconds)	RMS Elevation Error (degrees)	RMS Elevation Rate Error (degrees)
Noiseless	2.0	.001	.0003
	5.0	.0002	.002
	10.0	.003	.008
10 db	2.0	.097	.092
	10.0	.045	.012
5 db	2.0	.184	.176
	5.0	.135	.056
	10.0	.108	.028
0 db	2.0	.413	.384
	10.0	.289	.051

TABLE II

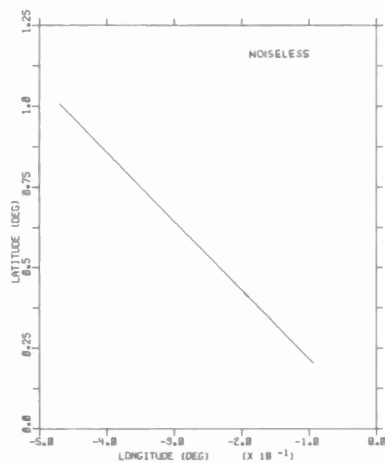
RMS Errors of Elevation and Elevation Rate  
for Different Elevation Smoothing Intervals  
and Signal-to-Noise Ratios

Signal-to-Noise Ratio	Azimuth Smoothing Interval (seconds)	RMS Azimuth Error (degrees)	RMS Azimuth Rate Error (degrees)
Noiseless	2.0	.00001	.00001
	5.0	.00001	.00001
	10.0	.00002	.00004
10 db	2.0	.102	.110
	10.0	.040	.010
5 db	2.0	.194	.198
	5.0	.138	.069
	10.0	.091	.019
0 db	2.0	.423	.453
	10.0	.187	.044

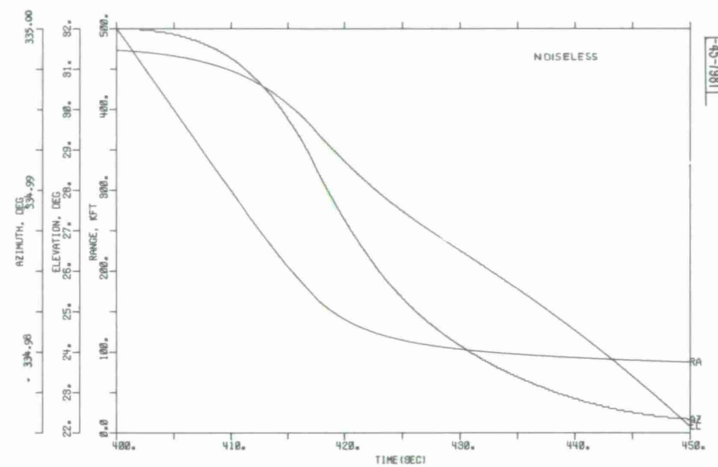
TABLE III

RMS Errors of Azimuth and Azimuth Rate  
for Different Elevation Smoothing Intervals  
and Signal-to-Noise Ratios

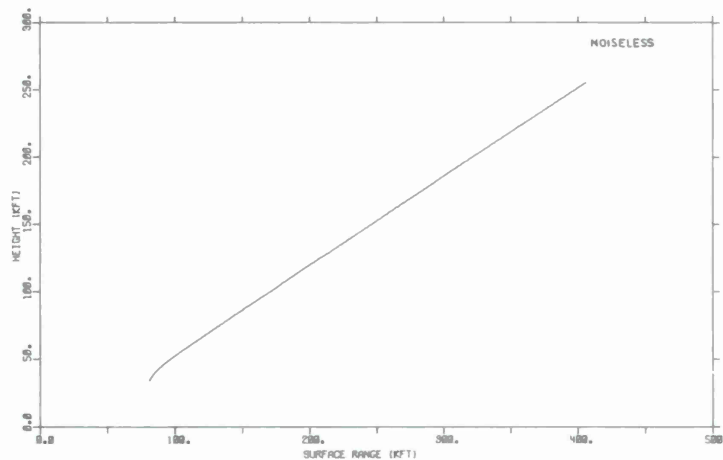




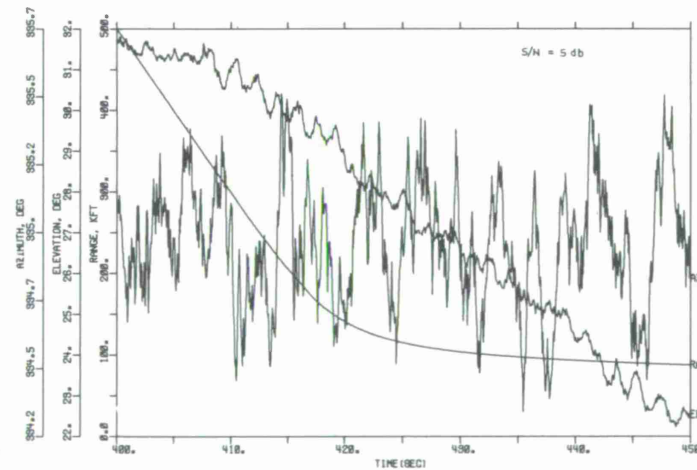
LATITUDE VS LONGITUDE



RANGE, ELEVATION, AND AZIMUTH VS TIME



HEIGHT VS SURFACE RANGE



RANGE, ELEVATION, AND AZIMUTH VS TIME

Fig. 1. Raw trajectory data vs. time for the noiseless and 5 db cases.



FITTING INTERVAL (sec)

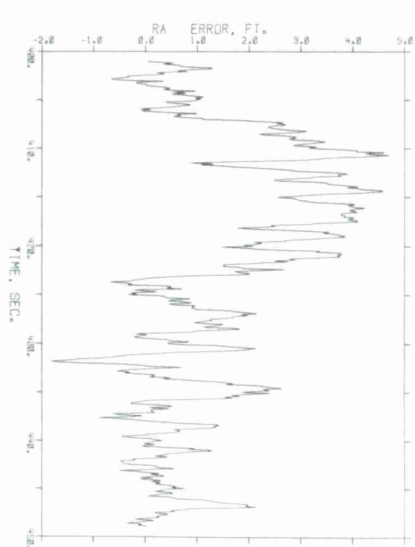
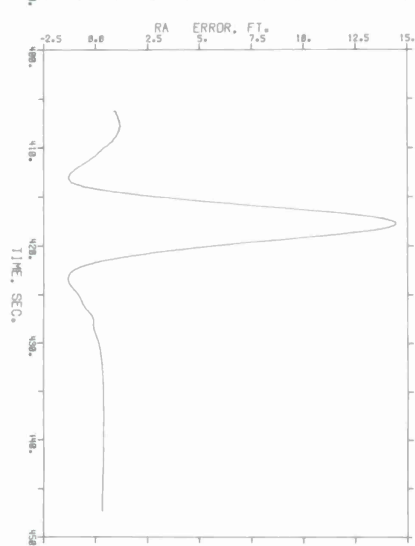
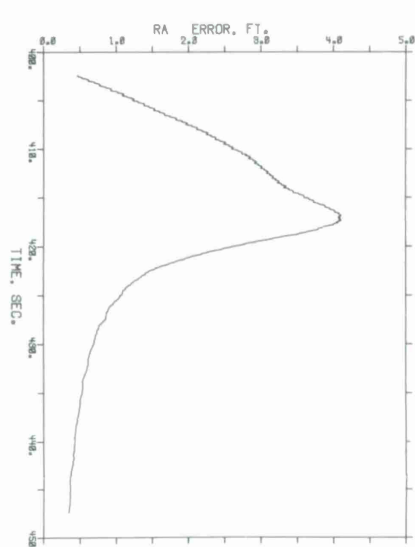
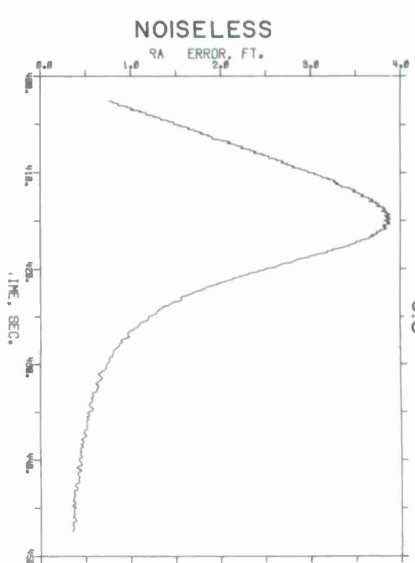
1.0

1.5

2.0

4.0

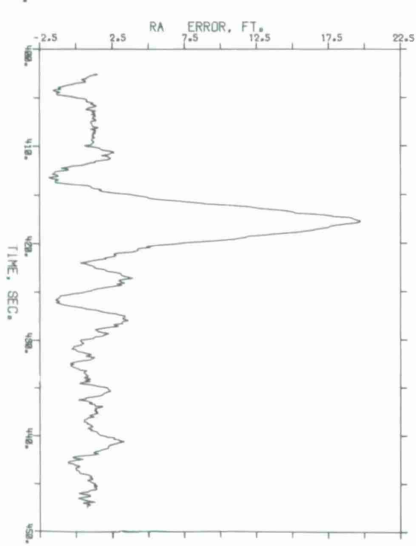
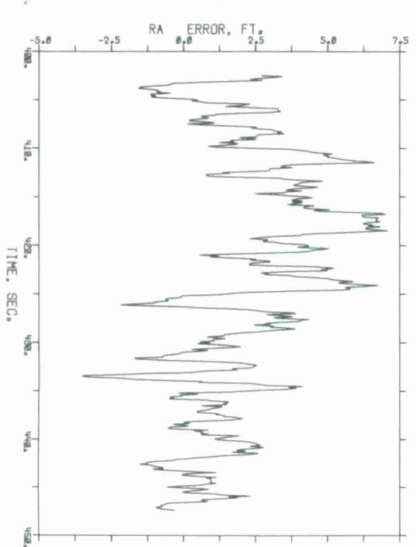
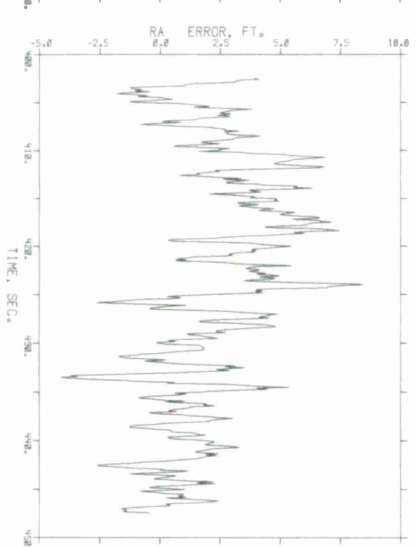
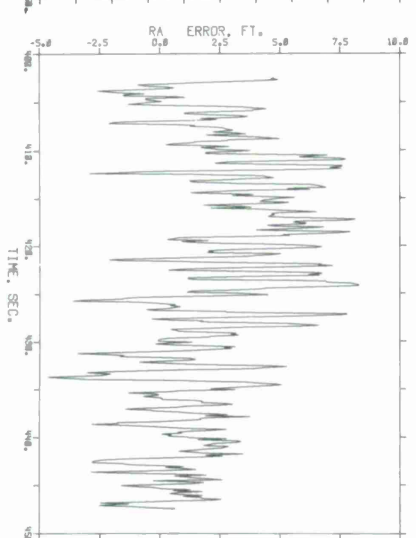
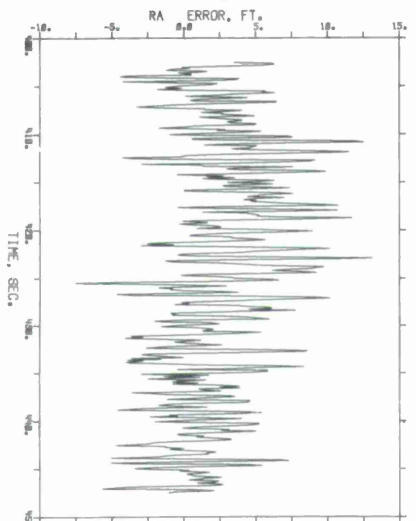
4-5-7972



SIGNAL-TO-NOISE RATIO (db)

10

5



0

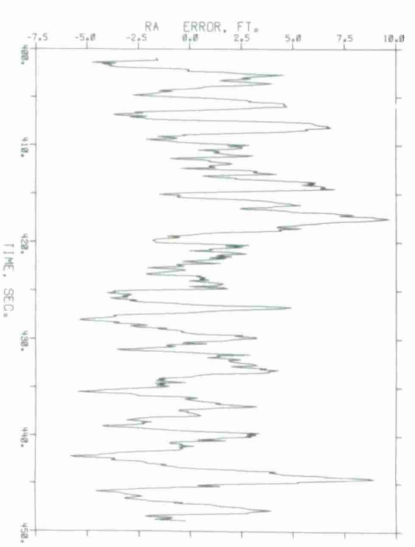


Fig. 2. Range error vs. time for different signal-to-noise ratios and smoothing intervals.



FITTING INTERVAL (sec)

-45-7977

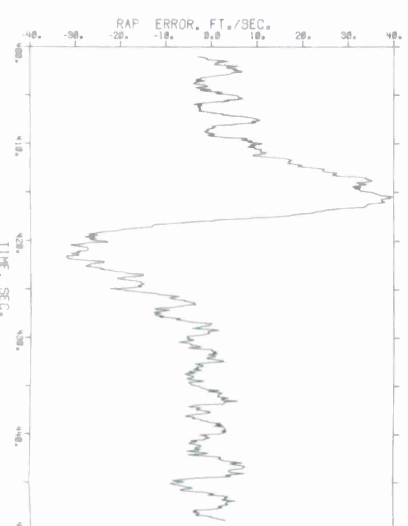
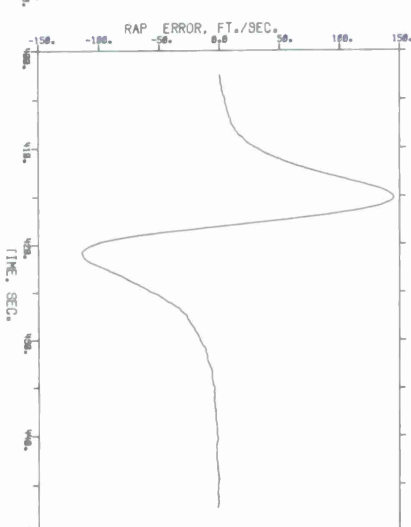
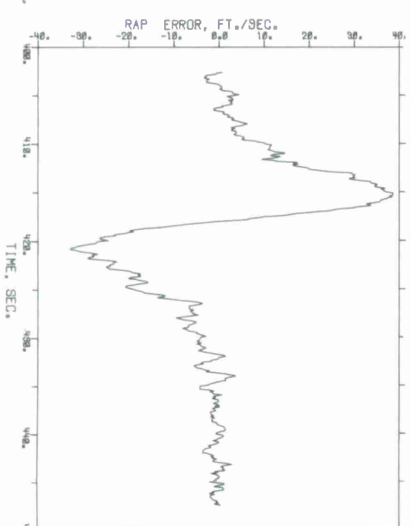
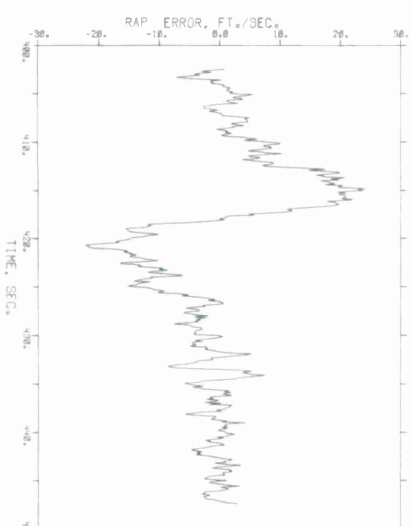
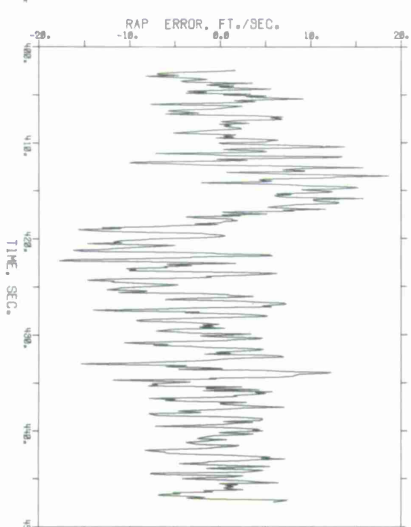
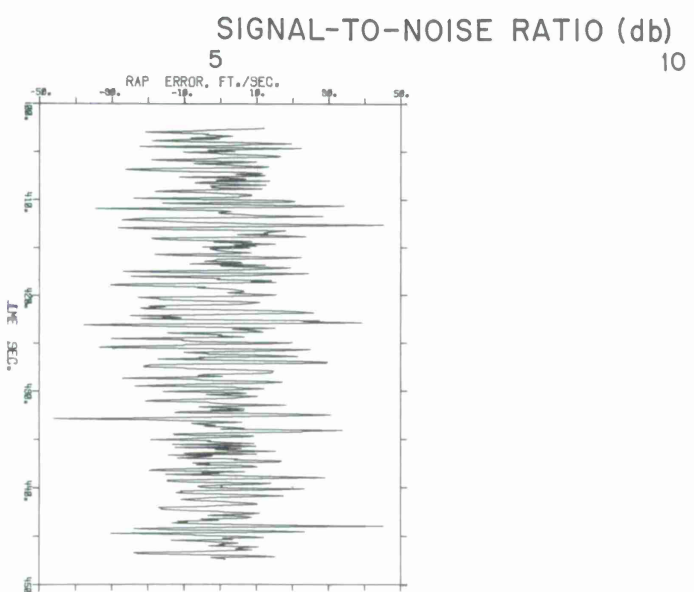
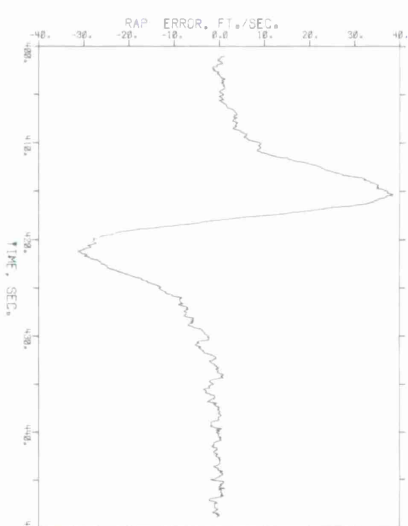
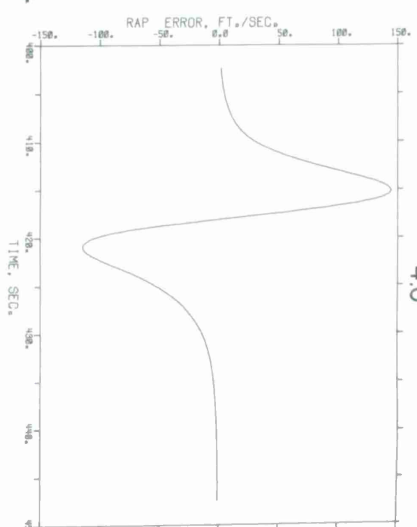
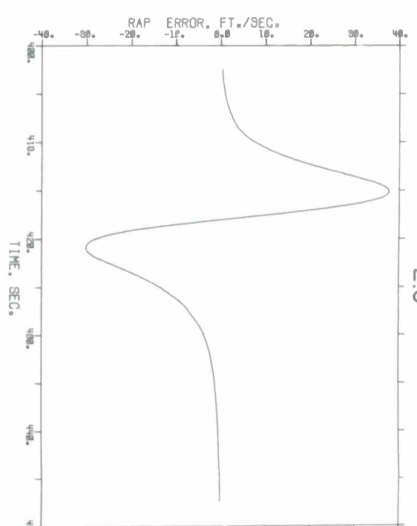
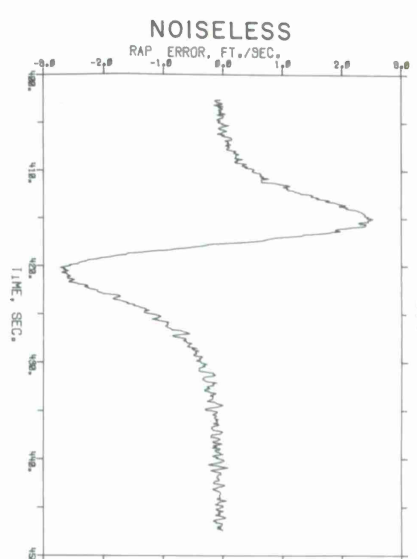
0.5

1.0

1.5

2.0

4.0



0

Fig. 3. Range rate error vs. time for different signal-to-noise ratios and smoothing intervals.



FITTING INTERVAL (sec)

45-7979

1.0

1.5

2.0

4.0

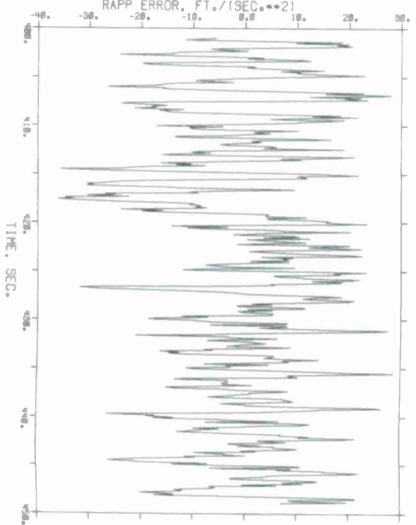
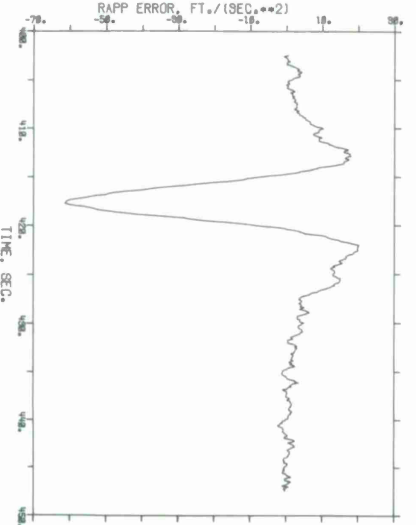
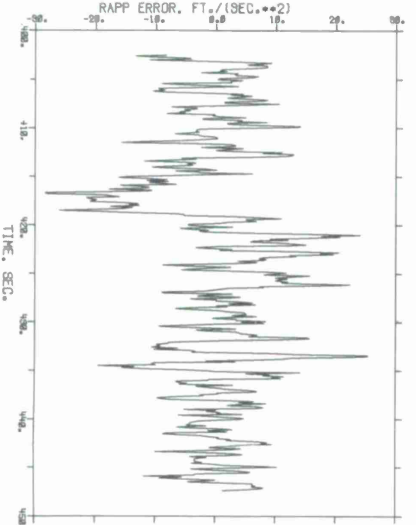
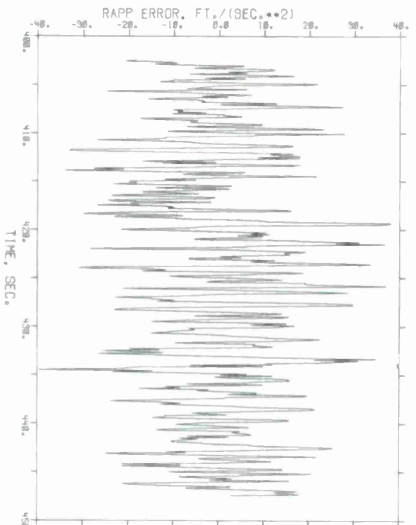
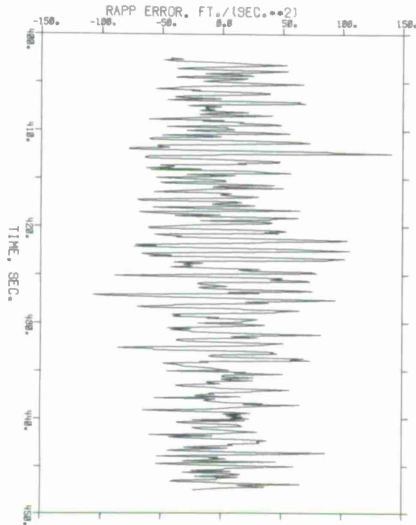
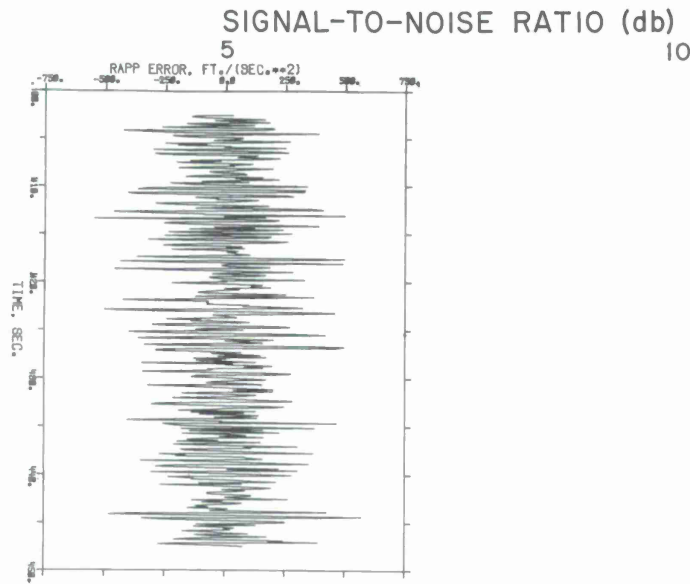
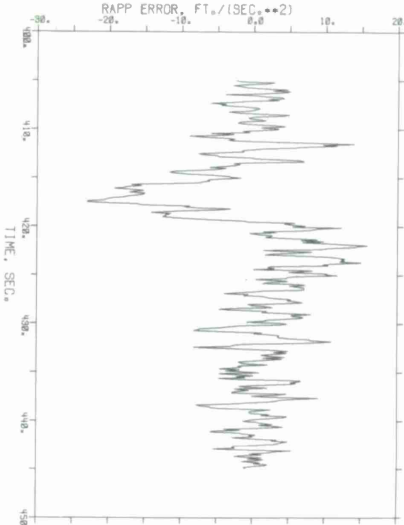
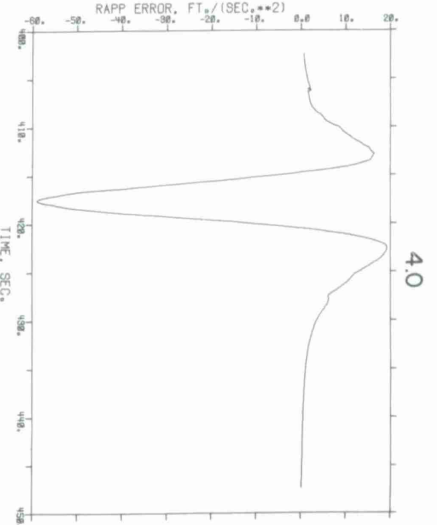
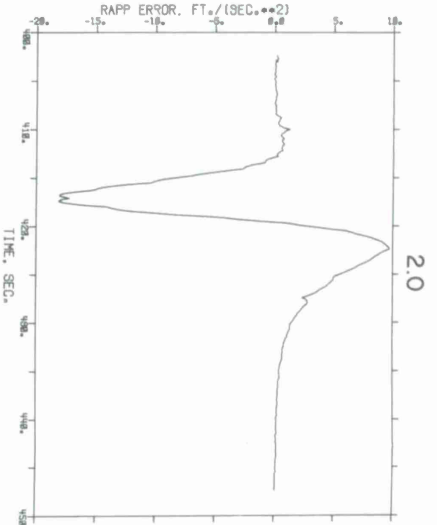
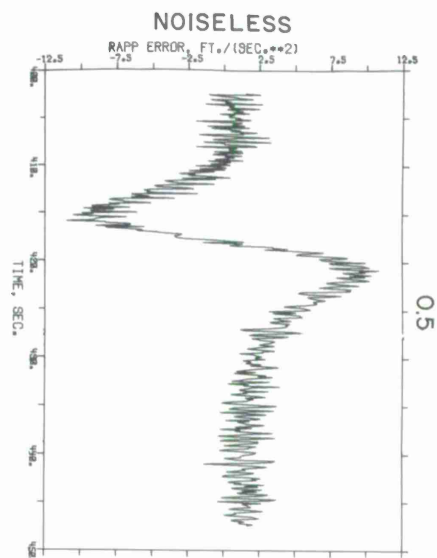


Fig. 4. Range acceleration error vs. time for different signal-to-noise ratios and smoothing intervals.

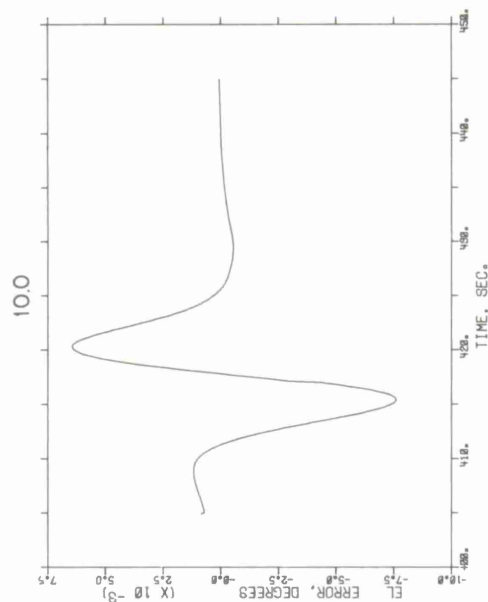
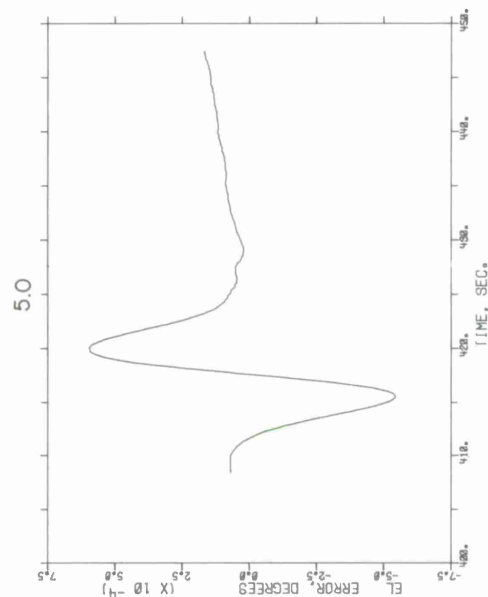




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FITTING INTERVAL (sec)

2.0



NOISELESS

SIGNAL-TO-NOISE RATIO (db)

20

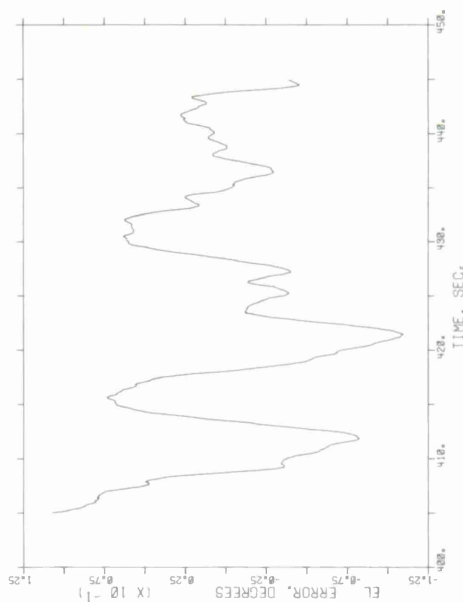
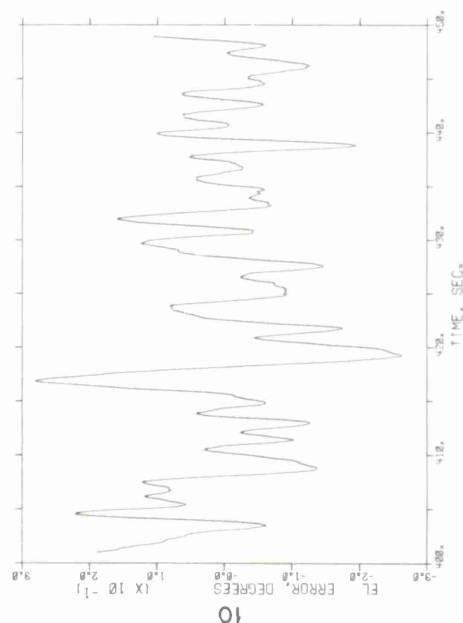


Fig. 5. Elevation error vs. time for different signal-to-noise ratios and smoothing intervals.

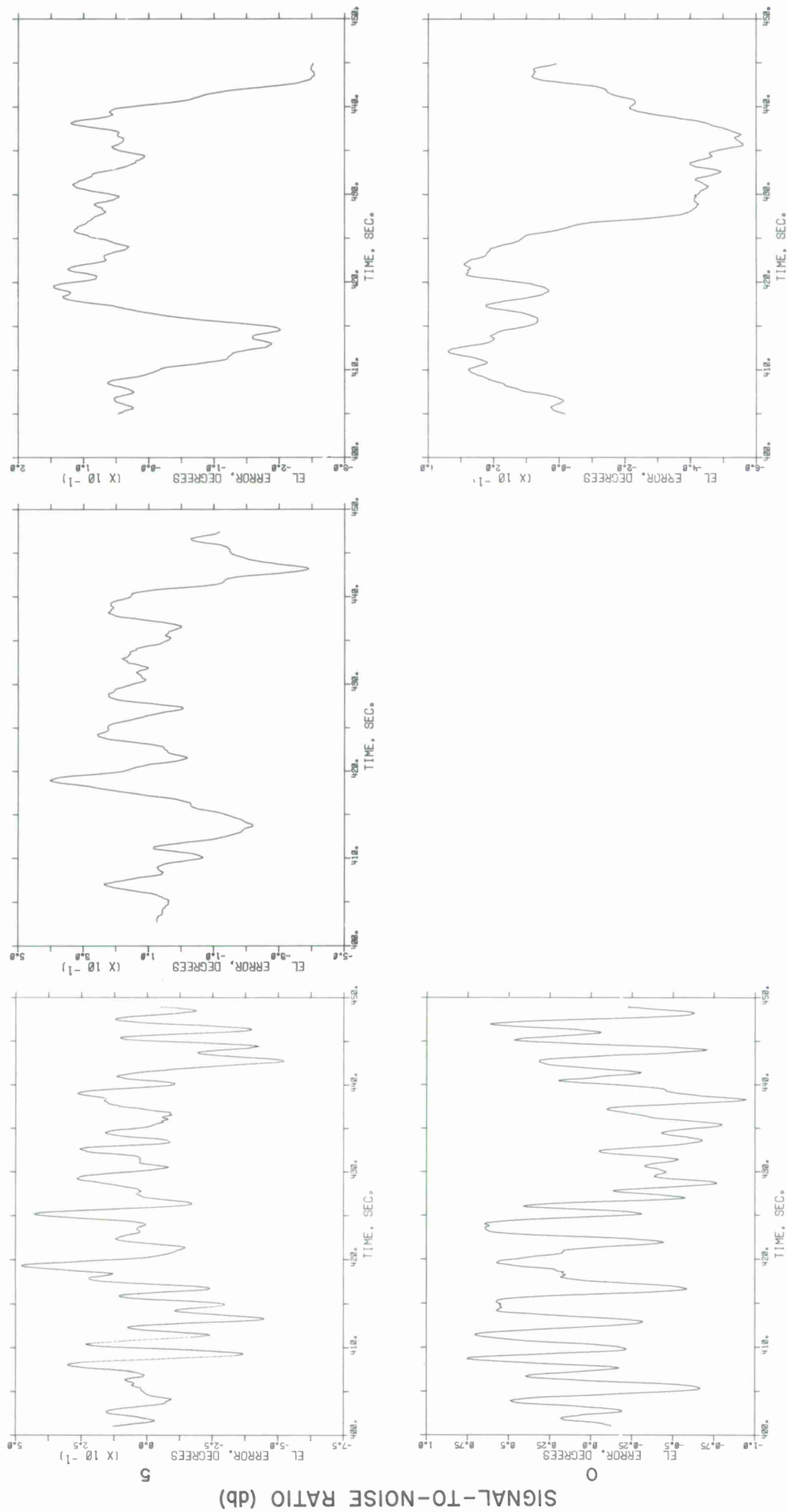
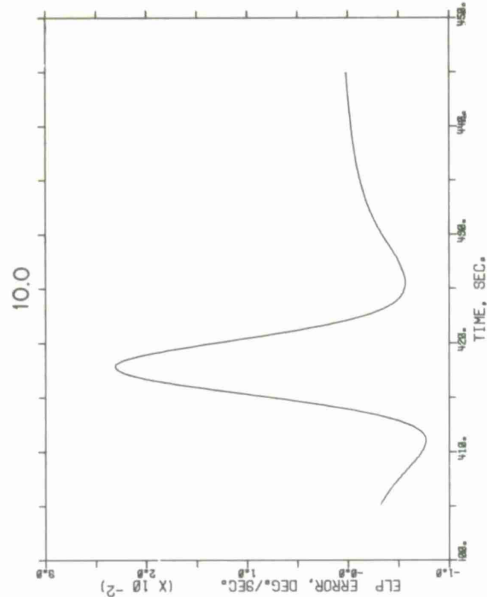
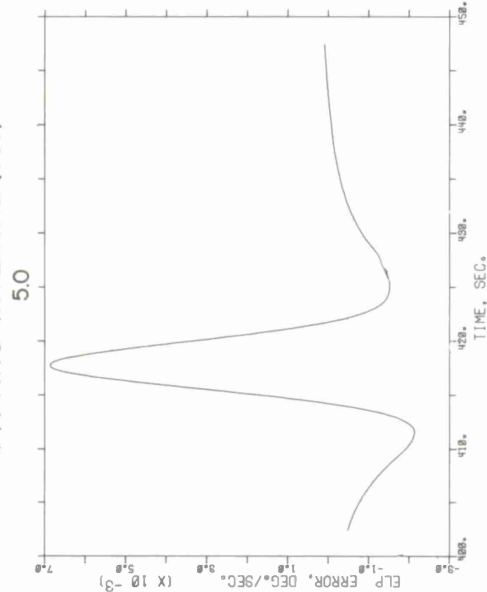


Fig. 5 continued

FITTING INTERVAL (sec)

2.0



NOISELESS

SIGNAL-TO-NOISE RATIO (db)

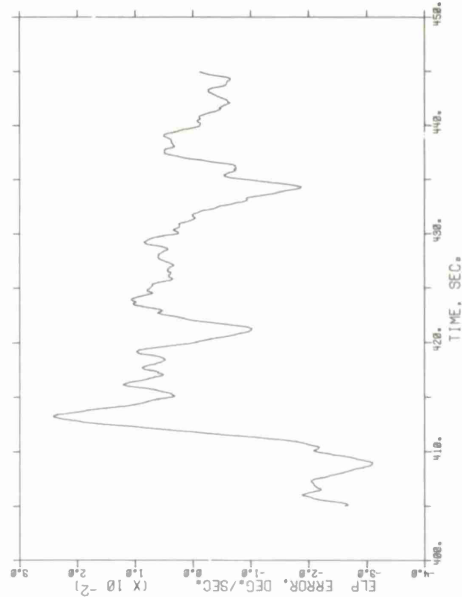
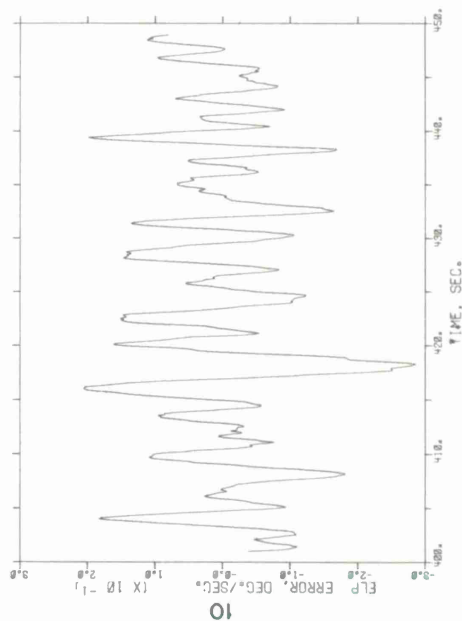


Fig. 6. Elevation rate error vs. time for different signal-to-noise ratios and smoothing intervals.

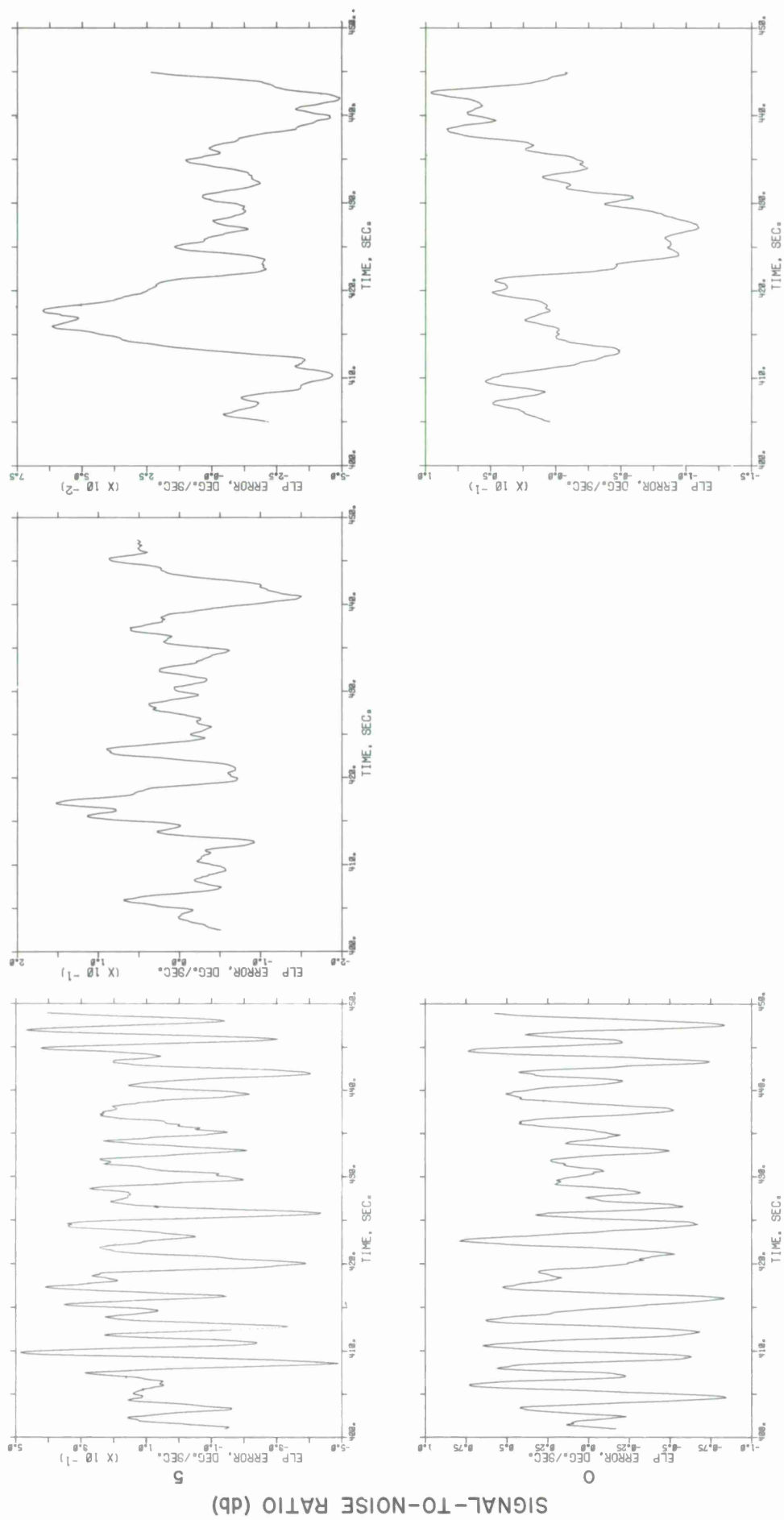
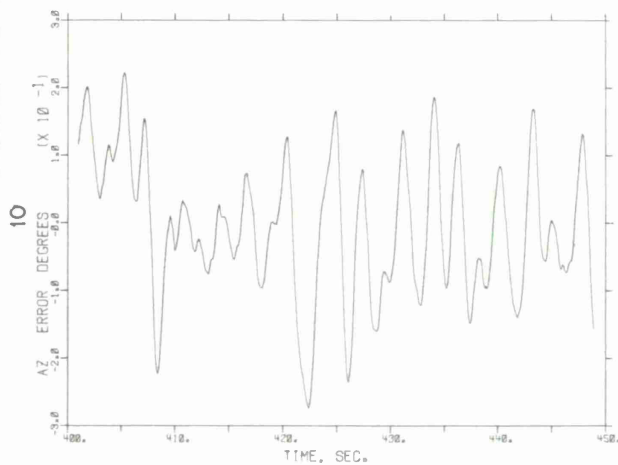


Fig. 6 continued

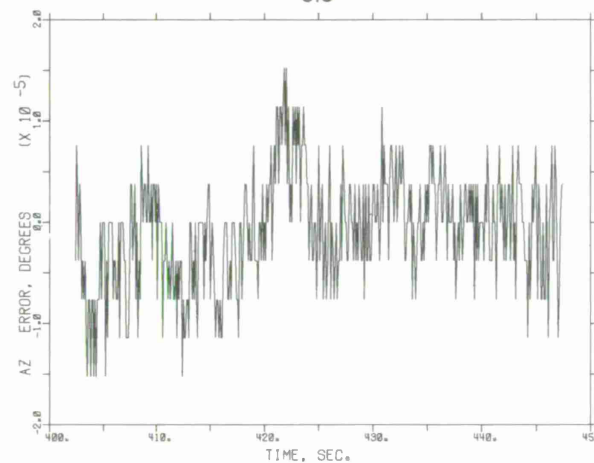
SIGNAL-TO-NOISE RATIO (db) NOISELESS



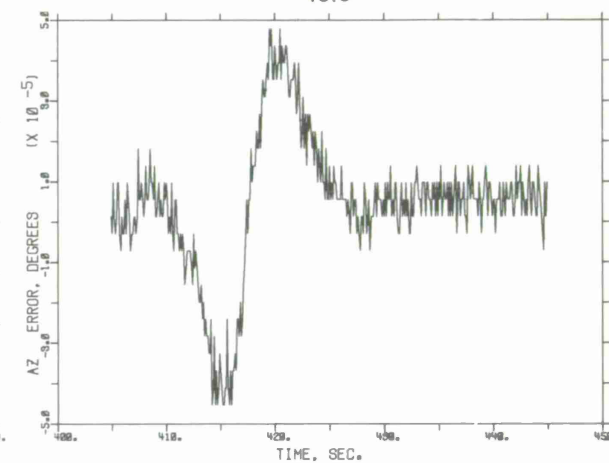
2.0

FITTING INTERVAL (sec)

5.0



10.0



-45-7973

Fig. 7. Azimuth error vs. time for different signal-to-noise ratios and smoothing intervals.

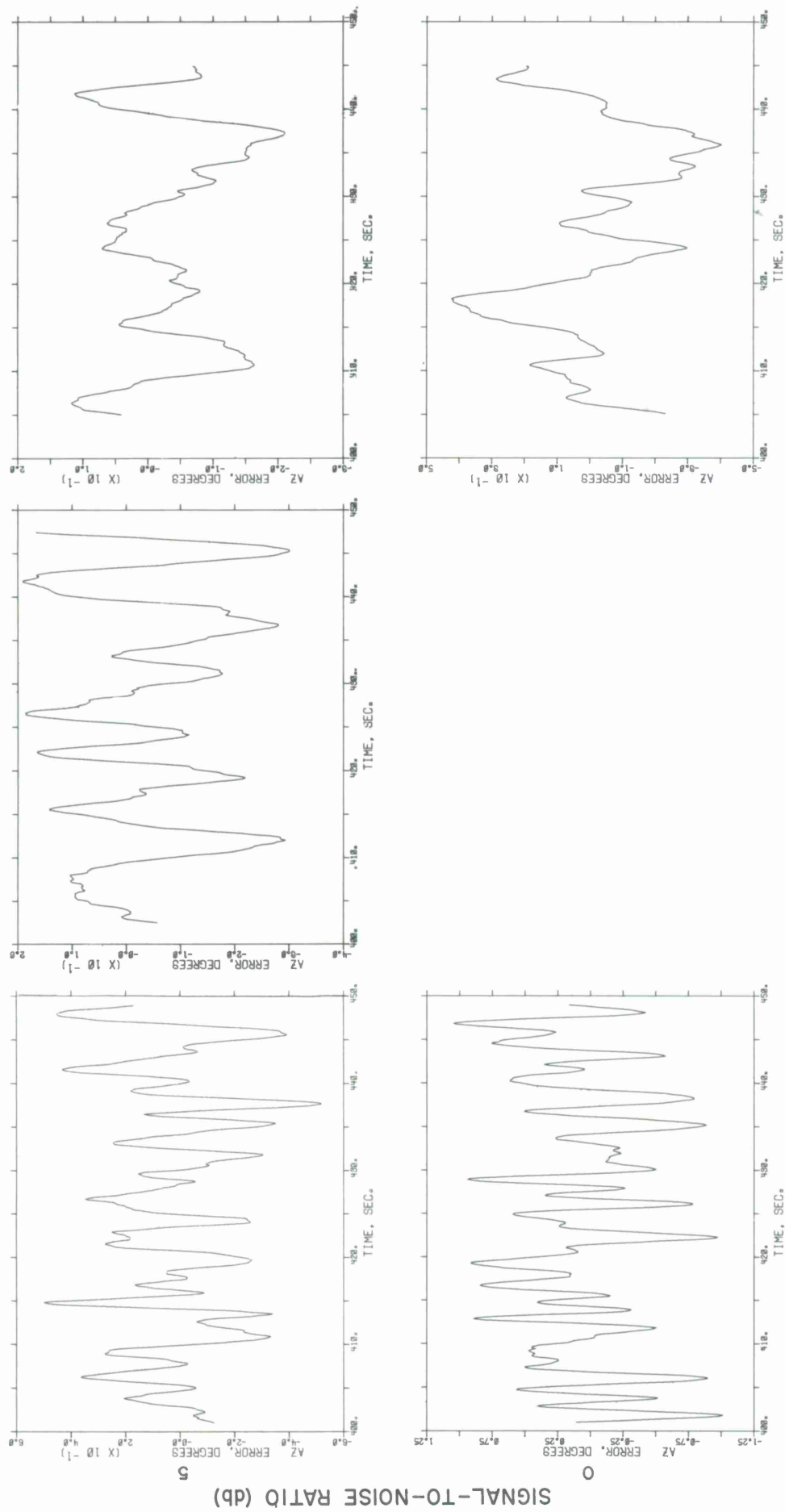
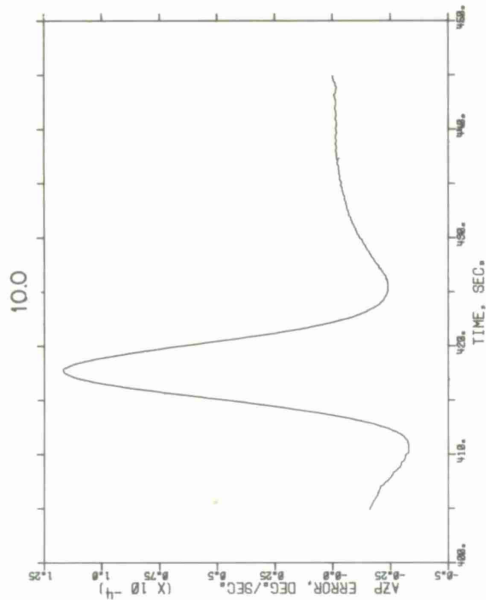
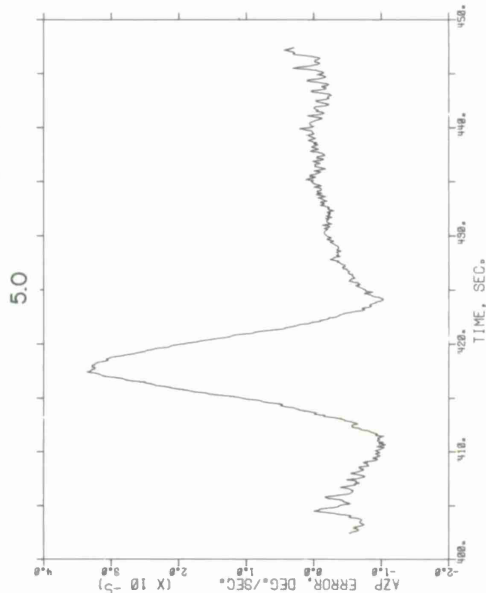


Fig. 7 continued

FITTING INTERVAL (sec)

2.0



SIGNAL-TO-NOISE RATIO (db)

26

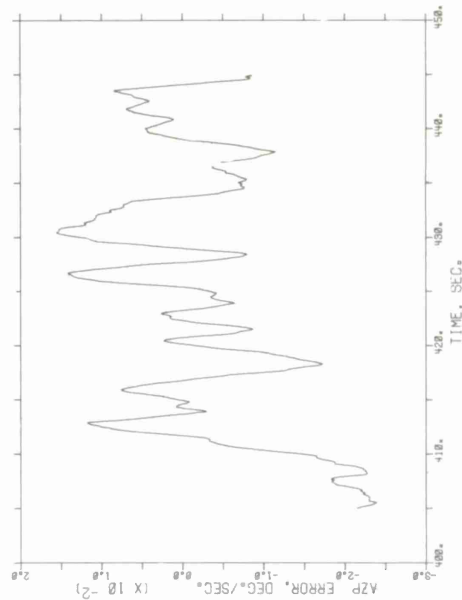
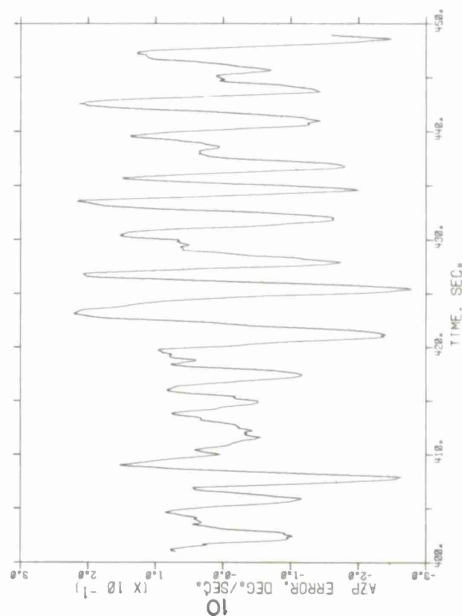
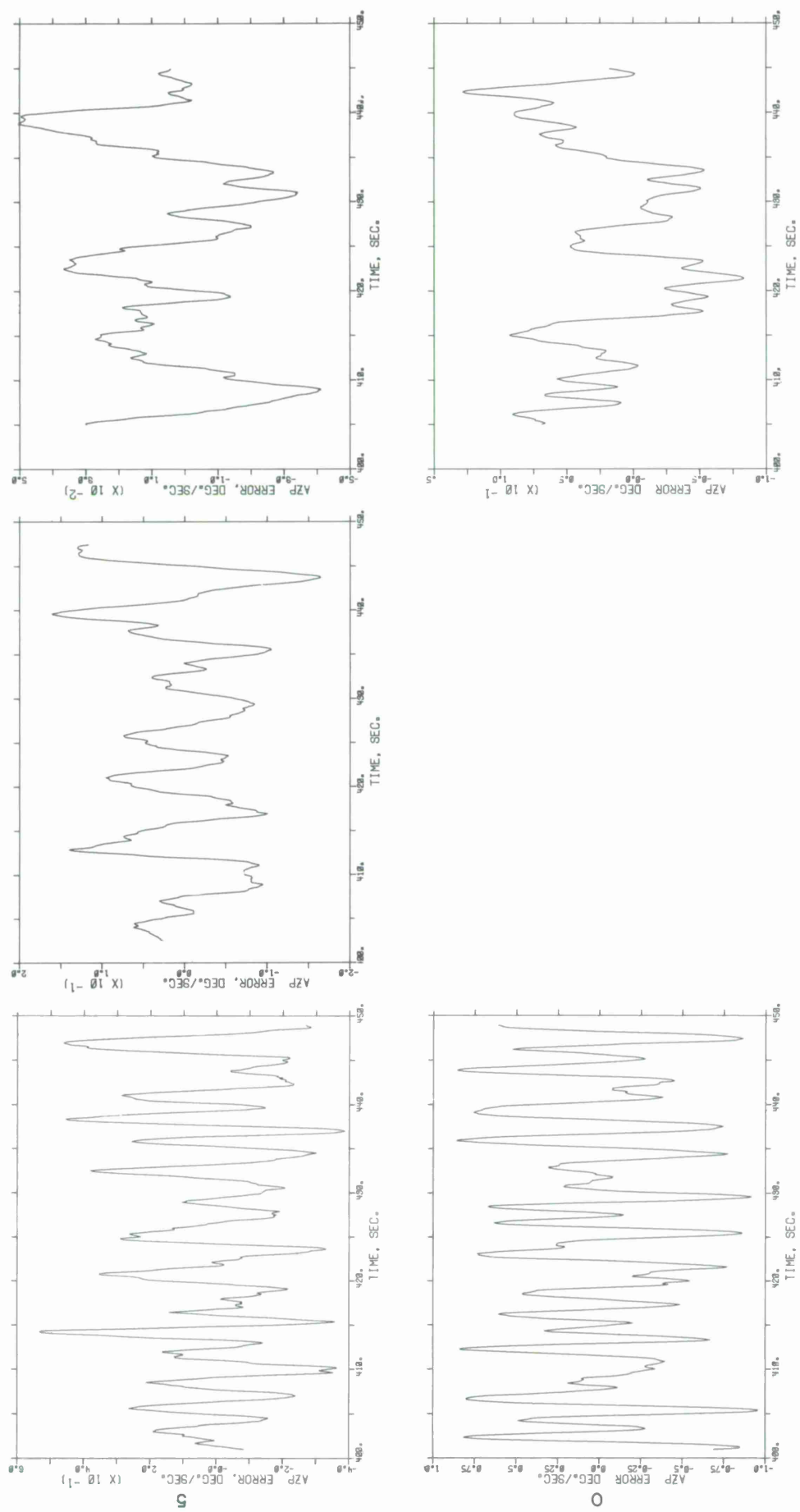


Fig. 8. Azimuth rate error vs. time for different signal-to-noise ratios and smoothing intervals.





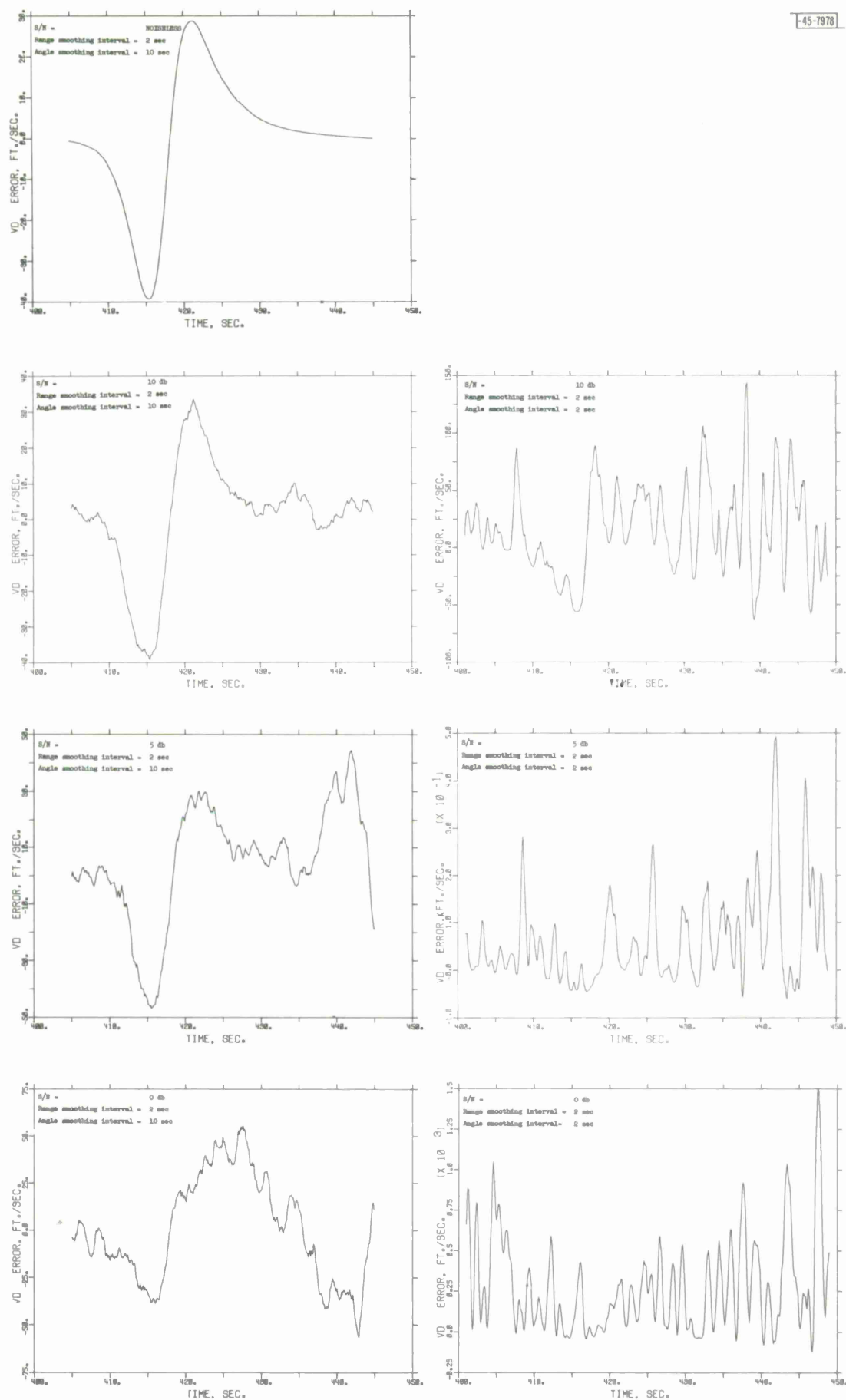


Fig. 9. Drag velocity error vs. time for different smoothing intervals and signal-to-noise ratios.

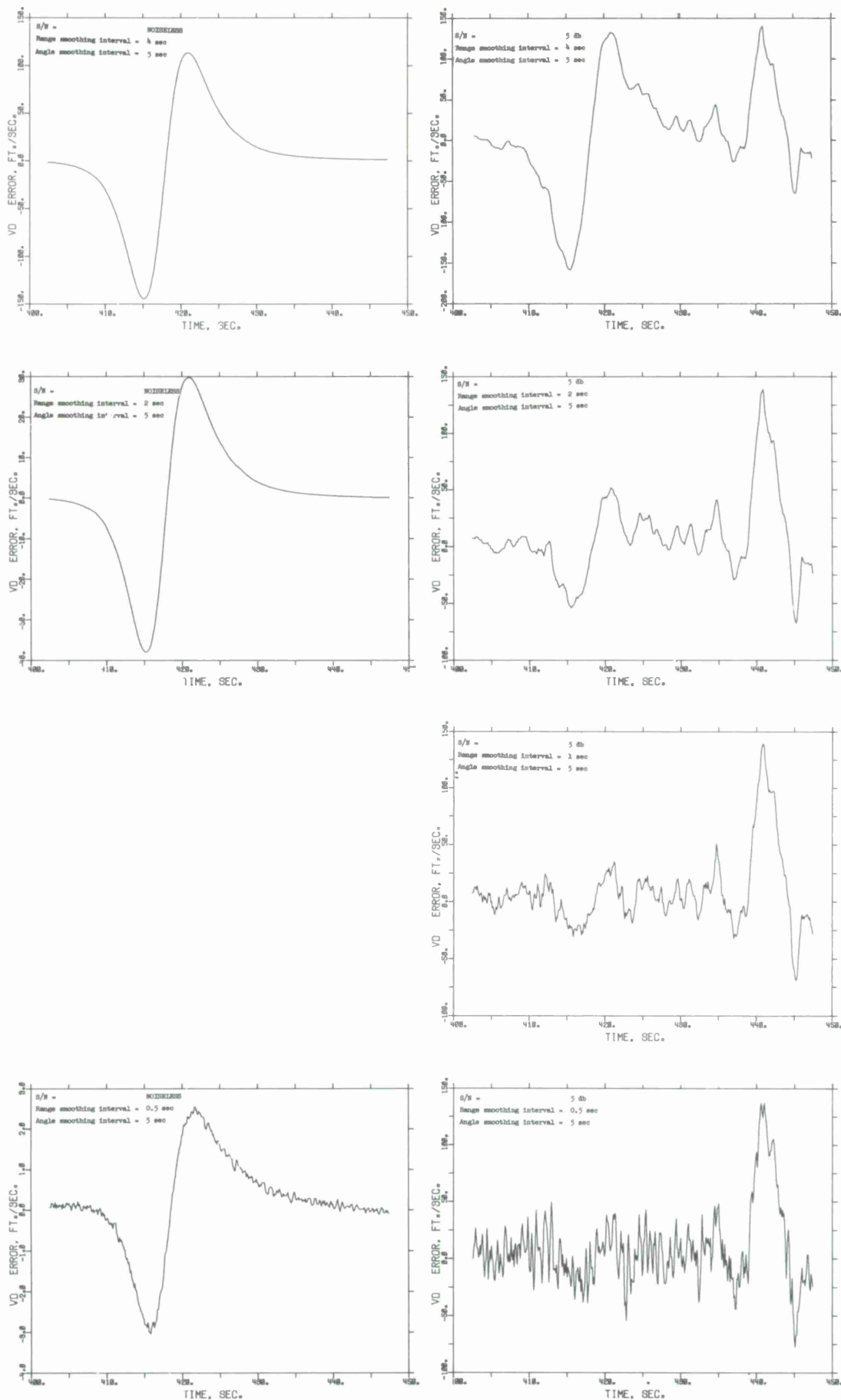


Fig. 9 continued

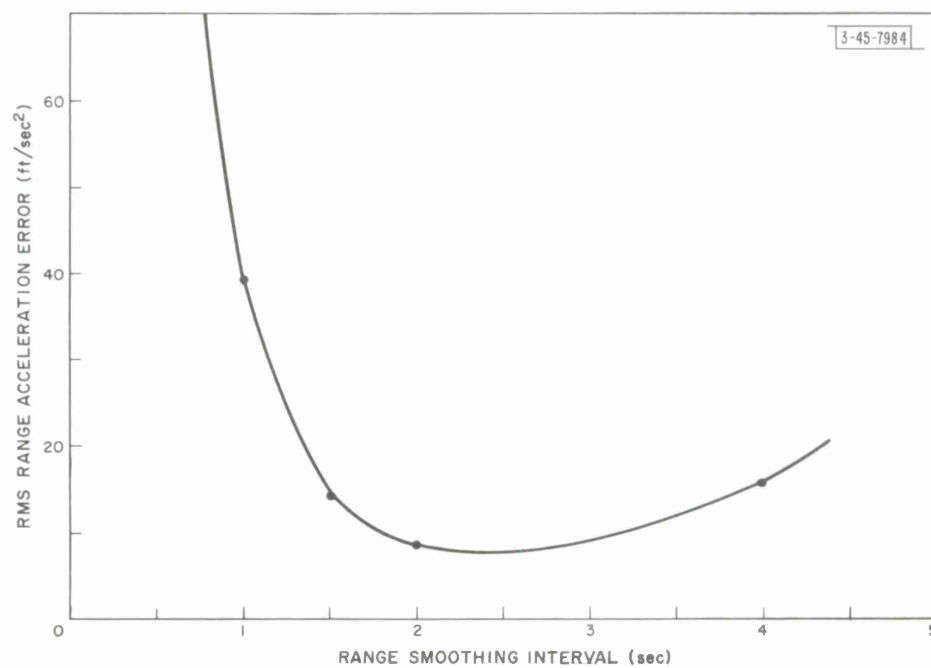
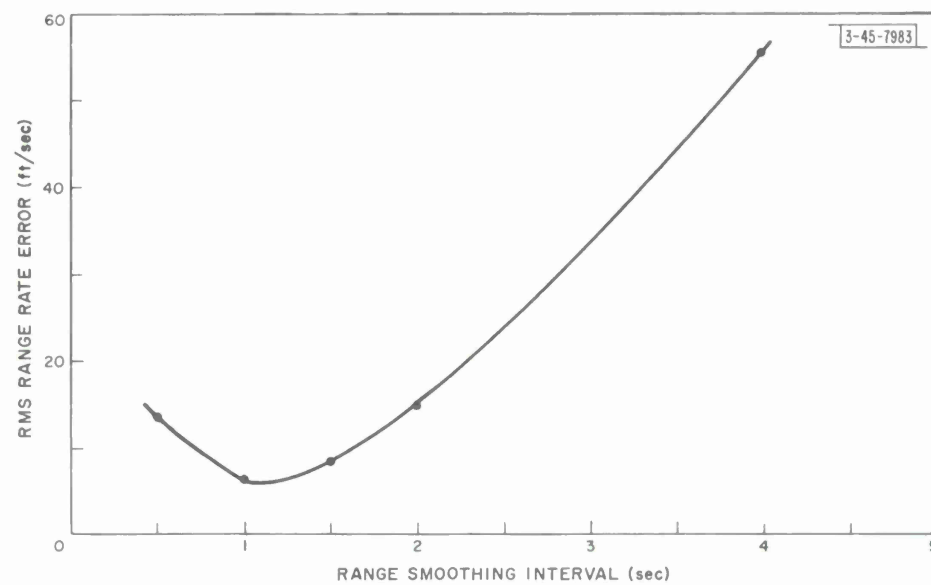
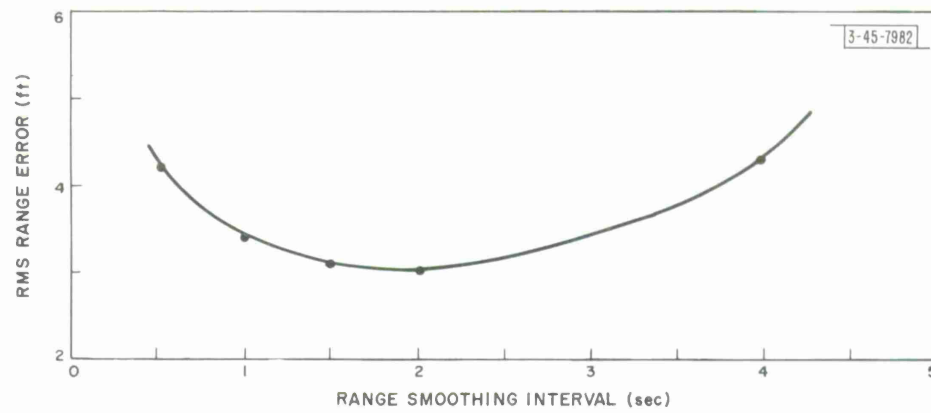


Fig. 10. RMS range errors vs. smoothing interval for the  $S/N = 5$  db case.

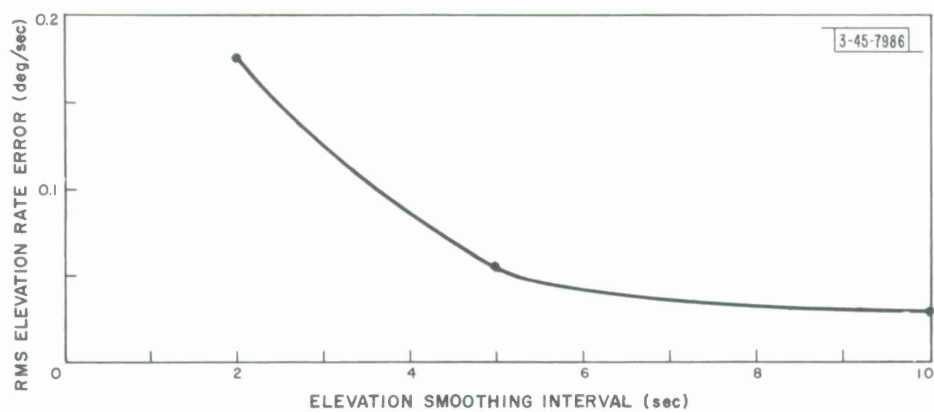
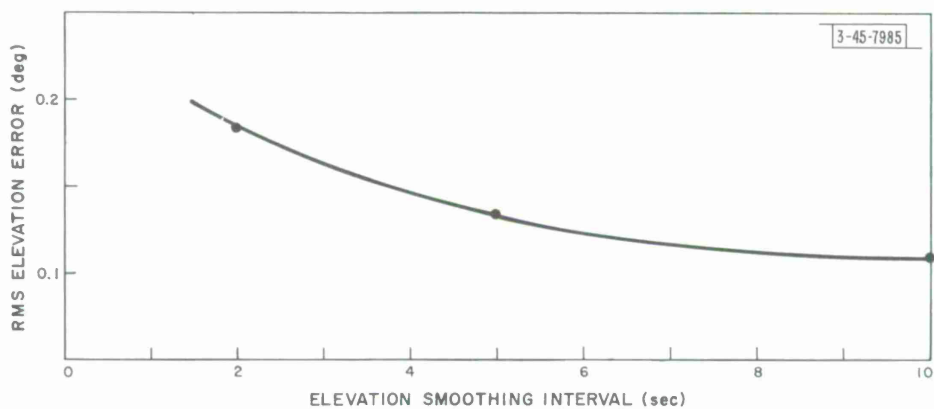
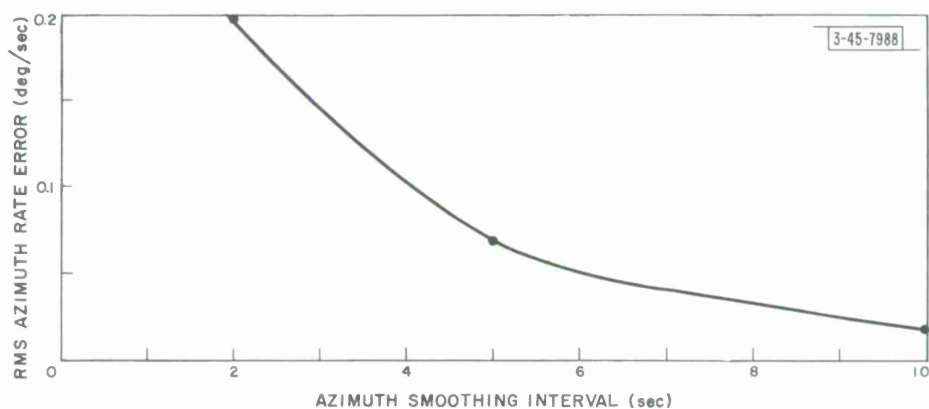
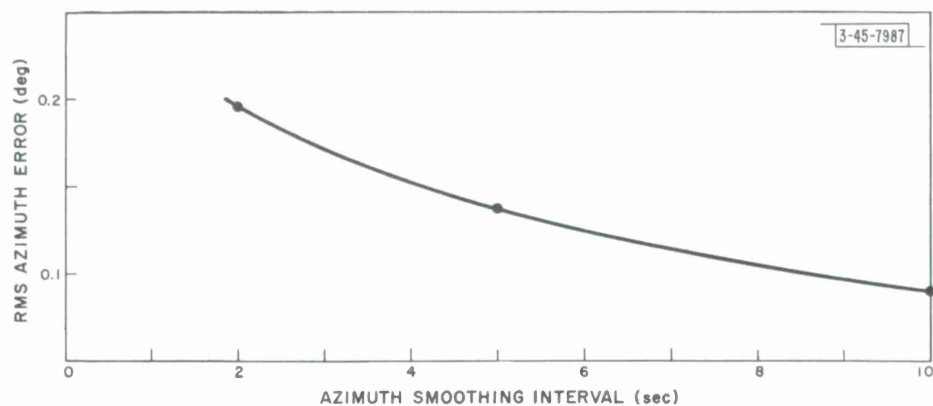
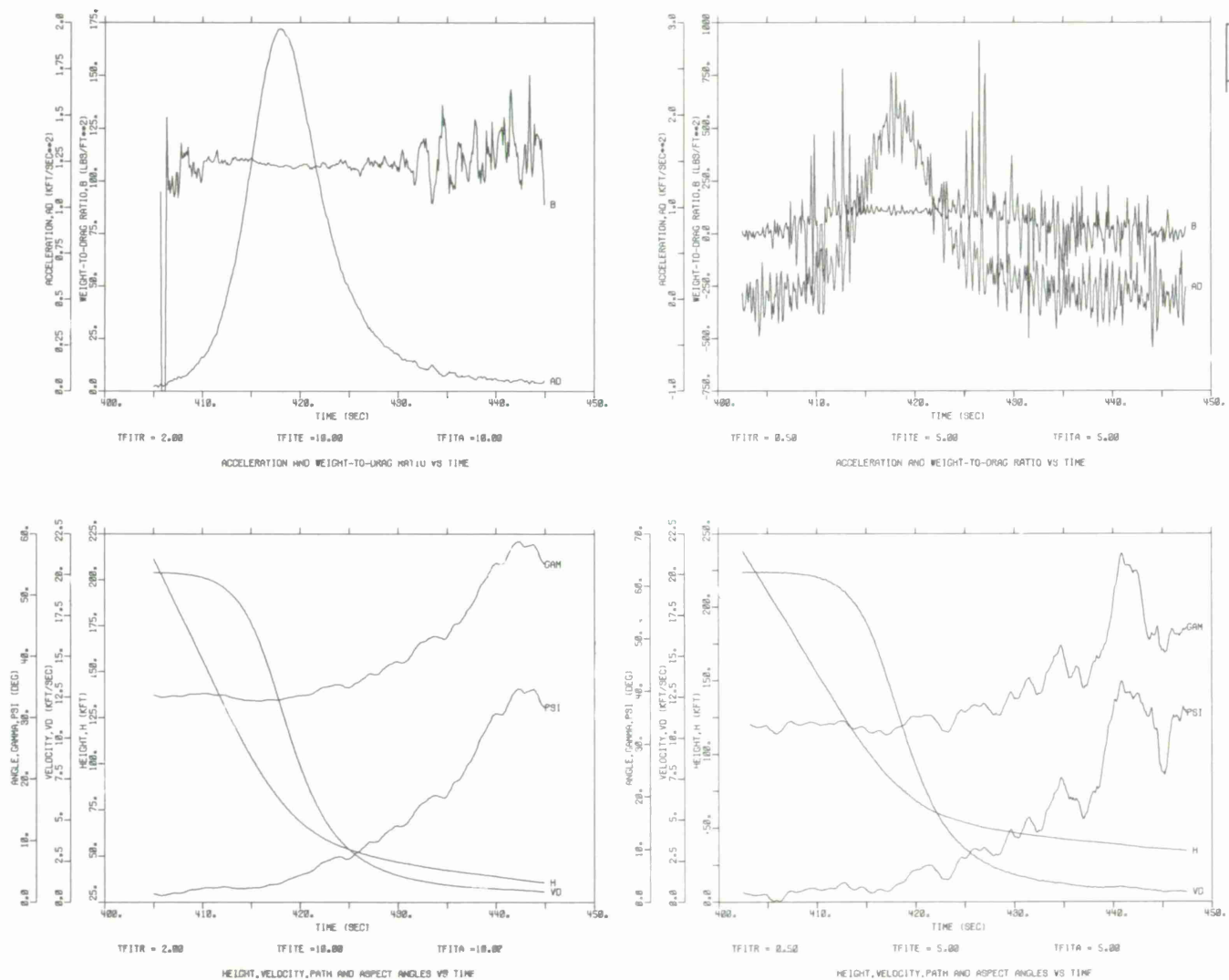


Fig. 11. RMS angle errors vs. smoothing interval for the  $S/N = 5$  db case.



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		2b. GROUP  n/a	
3. REPORT TITLE  Smoothing and Processing of Simulated AMRAD Trajectory Data			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)  Group Report			
5. AUTHOR(S) (Last name, first name, initial)  Bertolini, A. , Catalano, S.F.			
6. REPORT DATE  January 1965		7a. TOTAL NO. OF PAGES  35	7b. NO. OF REFS  0
8a. CONTRACT OR GRANT NO.  AF19(628)500  b. PROJECT NO.  c.  d.		9a. ORIGINATOR'S REPORT NUMBER(S)  ESD-TDR-65-33	
		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)  GR-1965-7	
10. AVAILABILITY/LIMITATION NOTICES  Qualified requesters may obtain copies from DDC.			
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